An Experimental Technique for Design of Practical Wireless Power Transfer Systems

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***Abstract*— Wireless Power Transfer using magnetically coupled resonators has been widely used for biomedical and consumer electronics applications. However, typical analysis of wireless power transfer has focused on standard operating conditions (linear power source and fixed impedances $Z_s = Z_L = Z_0$) and is not applicable for practical applications. In this work we use experimentally obtained S-parameters to accurately model and analyze a WPT system driven by a linear or non-linear power source with arbitrary source impedance (Z_s) delivering power to an arbitrary load impedance (Z_L). Using this approach, we define a framework to evaluate the performance of WPT in terms of two metrics i.e. efficiency and power delivered to the load. The proposed technique can help designers accurately predict and analyze the behavior of WPT systems in practical applications.**

I. Introduction

Wireless power transfer (WPT) using Magnetically Coupled Resonators (MCRs) has evolved into a popular solution for tether-free operation and charging of electric vehicles, consumer electronics and implanted biomedical devices. This technique uses high quality factor (Q factor) resonators coupled via magnetic flux lines to efficiently transfer power from the transmit to the receive side.

MCRs have been extensively studied in literature [1]–[4]. However, previous work has primarily focused on design and analysis of WPT in standard operating conditions. For example, a standard linear power source (typically 50 Ω output impedance VNA or functional generator) is used to drive the transmit resonator and a predefined fixed load (typically 50 Ω [1]–[3]) is connected to the receive resonator. Using these fixed source and load impedances and equivalent circuit models for the resonators, the WPT system is designed, optimized and analyzed in an ideal environment. However, practical applications utilize a Power Amplifier as the power source at the transmit side and a rectifier on the receive resonator (to convert the received AC power into DC). The impedance values of power amplifiers and rectifiers are dynamic and vary with operating parameters such as power levels, frequency, voltage and load application. Furthermore, the performance of the resonators is affected by surrounding environment. For example in implanted applications, biological tissues cause additional losses and frequency detuning which degrades performance of MCRs. The simplified linear circuit models discussed in literature does not accurately account for these parameters and hence are not directly applicable to practical systems.

In this short paper, we propose to mitigate these challenges by using experimentally obtained S-parameters to analyze a WPT system driven by a source with arbitrary source impedance (Z_s) delivering power to an arbitrary load impedance (Z_L). Unlike circuit models, use of experimentally obtained S parameters takes into account the accurate behavior of the resonators. Using these S-parameters we analyze systems driven by both linear and non-linear (such as PA) power sources delivering power to arbitrary load. We also define a framework to quantify the performance of WPT in terms of two metrics i.e. efficiency and power delivered to the load. Experimental results are presented in Section IV to demonstrate the efficacy and validity of our approach.

II. Wireless Power Transfer Analysis

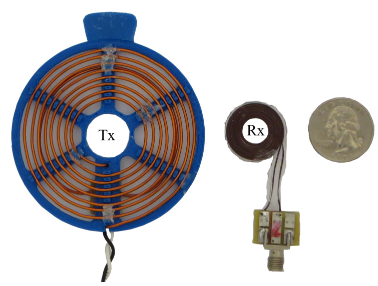

Fig. 1. Transmit and Receive MCRs used for experimental validation

Three different resonator topologies namely 2, 3 and 4 coil configurations have been demonstrated for Wireless Power Transfer. Although the analysis and evaluation presented in this work is applicable to all three resonator topologies, a 3 coil configuration designed and optimized for a small volume constraint implanted biomedical device is used in the experimental validation of the presented techniques. A 2 cm in diameter flat pancake coil and a 6.5 cm diameter loop and coil are used as the receive and transmit resonator respectively [5]. The resonators are tuned to operate in the 13.56 MHz ISM band. The MCRs used in this study are shown in Figure 1 and the summary of the coils is presented in Table I.

A. WPT Analysis with $Z_s = Z_L = Z_0$

Analysis of WPT system using equivalent circuit model based S-parameters (in a Z_0 system) has been extensively studied in literature [1]–[3]. However, in this work we will

TABLE I

MEASURED LUMPED MODEL PARAMETERS FOR THE MCRS

Transmitter		Receiver	
Outer Diameter	6.5 cm	Outer Diameter	2 cm
Wire	18 AWG Copper	Wire	24 AWG Copper
L_1, L_2	0.3 μH, 4.86 μH	L_3	2.16 μH
C_1, C_2	423.8 pF, 42.6 pF	C_3	63.7 pF
R_{p1}, R_{p2}	1.61 Ω, 4.86 Ω	R_{p3}	2.03 Ω
f_1, f_2	14 MHz, 13.6 MHz	f_3	13.57 MHz
Q_1, Q_2	16.5, 308.3	Q_3	90.6
k_{12}	0.37		

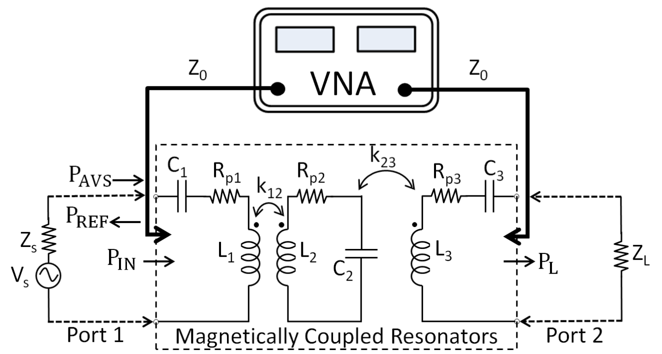

Fig. 2. A VNA Setup to extract S-parameters for Wireless Power Transfer

focus on experimentally obtained S parameters. Using a 50 Ω VNA (HP8753ES) and the setup shown in Figure 2, we measured the S-parameters of MCRs. Figure 3 shows the 3-D plot of the **experimentally measured** $|S_{21}|^2$ as the distance between the transmitter and the receiver is varied from 4 mm to 54 mm. It can be seen that for distances $\leq$ 9 mm, there exists an overcoupled region where high $|S_{21}|^2$ can be achieved [1]. The edge of this region is called the critical coupling point. For distances $\geq$ 9 mm, $|S_{21}|^2$ falls off rapidly with increase in distance. The high $|S_{21}|^2$ attained up to the critical coupling point is denoted by $|S_{21}|^2_{critical}$ = 0.89 in this case.

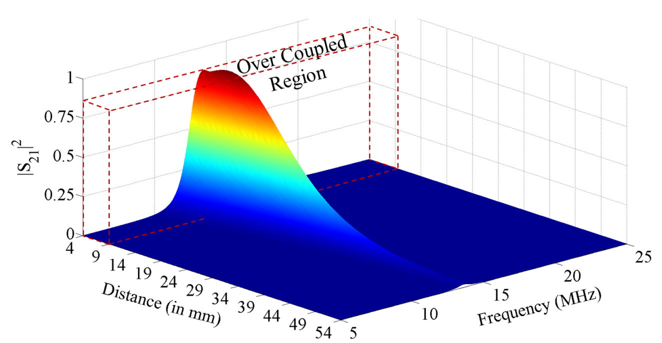

Fig. 3. Measured $|S_{21}|^2$ using a 50 Ω Vector Network Analyzer

B. WPT Analysis for arbitrary Z_s and Z_L

Although the study of MCRs with $Z_s = Z_L = Z_0$ provides valuable insight, it is not directly applicable for practical systems. In a typical WPT application, the receive coil is loaded by a rectifier and the input impedance of a rectifier is not equal to 50 Ω and varies with loading conditions such as operating frequency, voltage, load current and threshold drop of the diodes. Similarly, the input impedance of power amplifiers (switching amplifiers like class-E) is not necessarily 50 Ω. Hence, there is a strong motivation for analysis of WPT systems with arbitrary source impedance (Z_s) delivering power to an arbitrary load impedance (Z_L). Using the S-parameters experimentally obtained using a 50 Ω VNA we will analyze the generalized WPT systems. To this end, we define the source reflection coefficient (Γ_s) and load reflection coefficient (Γ_L):

$$\Gamma_s = \frac{Z_s - Z_0}{Z_s + Z_0} \qquad \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \tag{1}$$

Using Γ_s, Γ_L and S-parameters in a Z_0 system (obtained experimentally), the input reflection coefficient (Γ_{IN}) and output reflection coefficient (Γ_{OUT}) of a generalized WPT system can be defined as given below [6].

$$\Gamma_{IN} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \qquad \Gamma_{OUT} = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s} \tag{2}$$

Next, we define the Transducer Power Gain (G_T) as follows:

$$G_T = \frac{P_L}{P_{AVS}} = \frac{\textit{Power delivered to the load}}{\textit{Power available from source}} \tag{3}$$

$$= \frac{|S_{21}|^2 \left(1 - |\Gamma_s|^2\right)\left(1 - |\Gamma_L|^2\right)}{|1 - \Gamma_s\Gamma_{IN}|^2 \, |1 - S_{22}\Gamma_L|^2} \tag{4}$$

Transducer power gain (G_T) of the WPT system is its ability to deliver the power available from the source (P_{AVS}) to the load i.e. a larger G_T implies higher power delivered to the load and less strain on the power source/amplifier. This is critical for high end to end (including losses in the PA) efficiency of the system. For a Z_0 system, G_T simplifies to

$$G_T|_{Z_s = Z_L = Z_0} = |S_{21}|^2 \tag{5}$$

Similarly, Operating Power Gain (G_{OP}) can be defined as

$$G_{OP} = \frac{P_L}{P_{IN}} = \frac{\textit{Power delivered to the load}}{\textit{Power input to the network}} \tag{6}$$

$$= \frac{|S_{21}|^2 \left(1 - |\Gamma_L|^2\right)}{\left(1 - |\Gamma_{IN}|^2\right) |1 - S_{22}\Gamma_L|^2} \tag{7}$$

The operating power gain (G_{OP}) is a measure of the power lost in the resonators i.e. higher G_{OP} equals lower losses in the coils. For a Z_0 system, G_{OP} simplifies to

$$G_{OP}|_{Z_s = Z_L = Z_0} = \frac{|S_{21}|^2}{1 - |S_{11}|^2} \tag{8}$$

C. Power Amplifier Considerations

The input impedance of MCRs varies as a function of distance and can be computed using (2). The analysis in the previous section assumes that power available for source (P_{AVS}) is independent of load. Although this is is true for a linear power source, power amplifiers used in practical WPT systems seldom satisfy this requirement. Power amplifiers (PA) are designed using one of the following two techniques:

- $\boldsymbol{S_{22}}$ **match** wherein the output of the PA (Z_s) is conjugately matched to the load impedance, typically 50 Ω at lower drive levels. This translates into maximum power gain at relatively lower power levels of the output RF transistor [7].
- **Power match** wherein the load impedance is matched to optimal operating parameters of the output transistor of the PA at maximum output power. This translates into higher maximum output power from the PA [7].

For a PA designed using S_{22} match and operating within it's linear range (typically for low power levels), the analysis shown above is valid. However, in case of a power match PA (such as commonly used Class E switching amplifiers), the output power is a non-linear function of the load and should be determined using load pull measurements. In load pull measurements, the output port of the PA is attached to a variable load. The variable load is adjusted to match the different input impedance values of MCRs (as a function of distance) and the power at the output of the PA (P_{IN}) for these different load values is measured.

III. Evaluation of WPT system

Performance of a WPT system for a set of operating distances/scenarios can be quantified using the following metrics:

A. Power Delivered to Load (PDL)

For system driven by a linear power source, power delivered to the load can be determined from G_T.

$$PDL = G_T * P_{AVS} = G_T * \frac{V_s^2}{8\left\{Re\left(Z_s\right)\right\}} \tag{9}$$

However, for a non-linear power source since power available for source (P_{AVS}) is not constant, power delivered is computed using G_{OP} and P_{IN} (determined by load pull analysis).

$$PDL = G_{OP} * P_{IN} \tag{10}$$

B. Efficiency (η)

Efficiency (η) of the MCRs takes into account the power lost in the coils and can be expressed using G_{OP}.

$$\eta = \frac{P_L}{P_{IN}} = G_{OP} \tag{11}$$

Note that the efficiency shown in (11) takes into account only the losses in the coils and does not accommodate losses in the power source and the rectifier/power convertors on the receive side. For a WPT system it is necessary to simultaneously satisfy both *PDL* and η requirements. For example, ensuring high efficiency but not delivering sufficient power to the load is unacceptable. Similarly, delivering sufficient power to the load at low efficiency is undesirable due to excessive heating of the resonators and the surrounding environment (critical in biomedical applications due to presence of tissues).

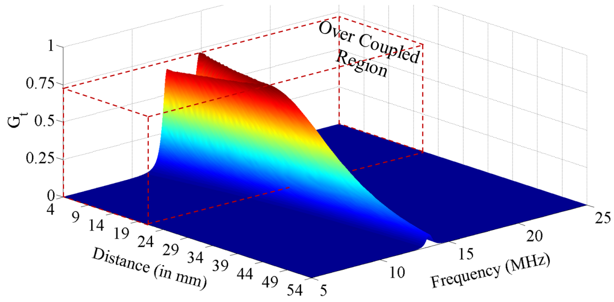

Fig. 4. Expected power delivered (normalized) to R_l load

IV. Validation and Experimental Results

Consider a WPT system driven by a $50\,\Omega$ linear power amplifier and loaded by a rectifier on the receive side. For low power applications such as ECoG recording, the input impedance of rectifier is the range of 1-10 kΩ [4]. In order to maintain a high Q factor receive coil, the rectifier is connected in parallel to the tuning capacitor (C_3). The parallel RC network can be transformed into a resistor (R_l) in series with C_3 to correspond to the circuit model shown in Figure 2. For our experiments, we model the rectifier load by a 6.3 Ω (R_l) resistor in series with the tuning capacitor C_3 .

For this practical system, the transducer power gain (analogous to $|S_{21}|^2$ in a Z_0 system) can be determined using (4) and is shown in Figure 4. The difference in the power delivered to a $50\,\Omega$ load and the rectifier (R_l) is evident from Figure 4 and 3. It can be seen that due to a non $50\,\Omega$ load, the critical coupling point has shifted to 22 mm (from 9 mm) and the maximum power point has dropped to 0.72 (from 0.85). A similar analysis can be performed to study the differences in the efficiency.

Please note that goal of this work is to outline analysis of a WPT system with arbitrary source and load impedance using experimental results. Derivation and optimization of $|S_{21}|^2_{critical}$, critical coupling point, η and *PDL* using MCR parameters (such as Q, k, Z_s, Z_L) is not the focus. However, it should be noted that using the proposed framework such an analysis can be undertaken and is the topic of future work.

To experimentally validate (9) and (11), Port 1 of the VNA (with $50\,\Omega$ source impedance) was connected to the transmit side and a 6.3 Ω load was connected to the receiver. The distance between the transmit and receive coils was varied in 2 mm increments and the input reflection coefficient (obtained using the VNA) and the power delivered to the load was measured. Figure 5 shows the variation of the input reflection with distance for a $50\,\Omega$ load and R_l (6.3 Ω) at the operating frequency, 13.56 MHz. It can be seen that the experimental input reflection coefficient values are in good agreement with the values expected from (2).

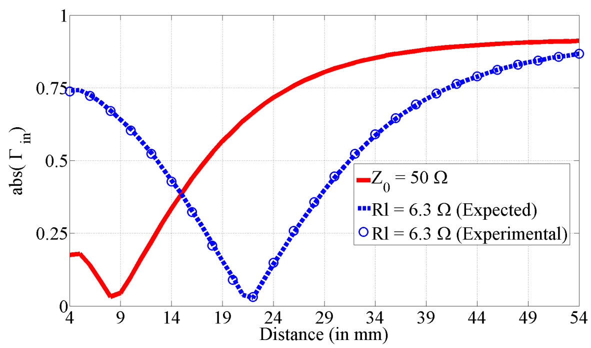

Fig. 5. Experimental validation for input reflection coefficient

Similarly, the transducer power gain (analogous to $|S_{21}|^2$ in a Z_0 system) can be experimentally validated. Figure 6 shows the variation in G_t (power delivered to load normalized to available power) with distance for a $50\,\Omega$ load and R_l(6.3 Ω) at 13.56 MHz. The experimental power data agree with theoretical values estimated using (4)).

To demonstrate the impact of a PA on WPT, a commercial PA (model 100W1000B by AR) was setup to deliver 200 mW to a $50\,\Omega$ load at 13.56 MHz. Load pull analysis of the PA was carried out by connecting a bi-directional coupler and a variable load to the output of the power amplifier. The bi-directional coupler was used to measure the output power (difference between forward and reflected power) of the PA as the load was varied. The variable load was adjusted to match the different input impedance values presented by the MCRs as distance between transmit and receive resonators is varied (This is clearly evident in Γ_{IN} shown in Figure 5). Using (10) and the output power of the PA (P_{IN}) measured using load pull analysis, the power delivered to load was analytically determined. Next, the power delivered to the load (R_l) by the PA as a function of the distance was experimentally measured. Figure 7 shows the experimental and theoretical results which are in good agreement with each other. The mismatch between

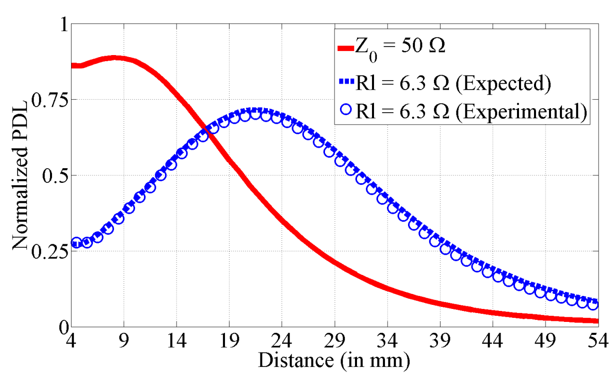

Fig. 6. Experimental validation for power delivered by a linear power source

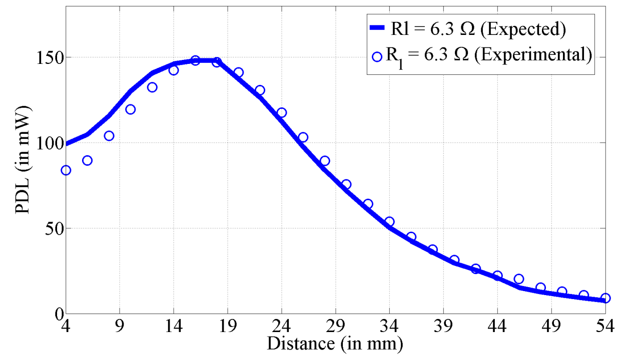

Fig. 7. Experimental validation for PA driven MCRs powering 6.3 Ω load

expected and experimental results for smaller distances can be attributed to high VSWR which resulted in miscalibration of the coupling factor of the directional coupler.

The significance of this analysis can be understood by noting the difference between Figure 6 and Figure 7. For MCRs driven by a linear power source (VNA), the maximum power is delivered at 22 mm (Figure 6) whereas when a PA powers the same MCRs, maximum power is delivered at 16 mm (Figure 7). Hence, the proposed analysis of PA driven MCRs is important to precisely model and optimize the efficiency and/or operating range of a practical WPT systems.

V. Conclusion and Future Work

In this work we have presented a technique for analysis of WPT system driven by linear or non-linear sources with arbitrary impedance delivering power to arbitrary load impedance using experimentally obtained S-parameters. We believe this approach can help the designers to accurately predict the behavior of WPT systems in practical applications. In the future, we will apply the proposed approach to optimize the design of MCRs and develop highly efficient and large operating range practical wireless power applications.

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